Assessment Results

Mechanical Engineering 341: Mechanics of Machinery

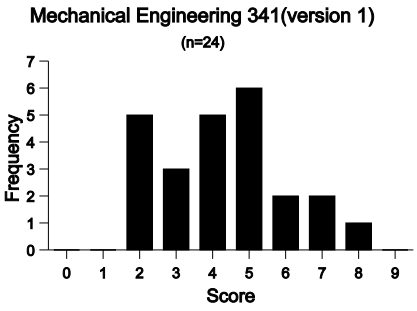
Fall 2001-02

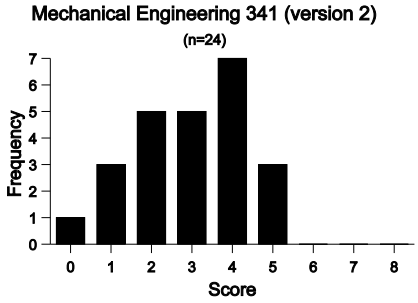
Forty-eight students took two versions of a free-response test in Mechanical Engineering 341 (Professor Jazar) during the first month of the Fall 2001 semester. The test was designed to see the extent to which students had quantitative skills required for success in the course. Graduate students from the Department of Mathematics graded the papers, recording information about steps students had taken when solving the problems. The graders also coded the degree of success achieved on each problem using the following rubric:

A Completely correct
B Essentially correct—student shows full understanding of solution and only makes a minor mistake (e.g., wrong sign when calculating a derivative or arithmetic error)
C Flawed response, but quite close to a correct solution (appears they could do this type of problem with a little review or help)
D Took some appropriate action, but far short of a solution
E Blank, or nothing relevant to the problem

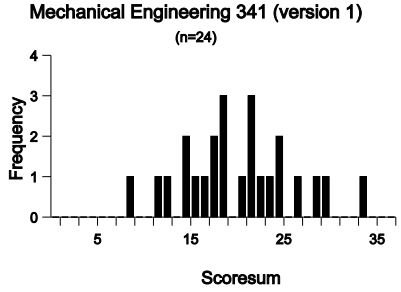
Corrected papers, along with suggested solutions to the problems, were returned to students. Summaries of the grader's coding are included on the attached copy of the test.

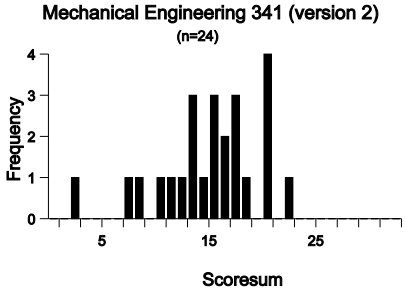
A test score was computed by awarding one point for each A or B code and zero points for each C, D, or E code. This score reflects the number of problems that each student had essentially or completely correct. The distributions of test scores are shown in these figures.





The second pair of charts gives the distribution of partial credit scores called scoresum (each problem was awarded 0-4 points, E=0 to A=4).





It appears that many students will need to review some mathematics covered on the test, since a majority were successful on fewer than half the problems. About a quarter of the students (14 of the 48) appear to have considerable difficulty with some of the material, achieving overall success on two or fewer of the eight problems.

The problems are ranked according the degree of success students achieved on each problem in the following table.

Degree of Success on Test Problems

%AB	%A	%C	No	Problem description
92%	92%	0%	V2#1b	Find equation of line through point parallel to given vector
88%	88%	0%	V1#2	Solve 3x3 system given corresponding matrix and inverse
75%	75%	0%	V1#7	Solve 2x2 matrix equation
75%	33%	0%	V1#4	Label graphs as functions of its first and second derivatives
67%	42%	0%	V1#5a	Find and integrate sum of vectors in plane
58%	58%	0%	V1#1	Use linear approximation to estimate value
58%	21%	21%	V2#2	Estimate derivatives from graph of function
46%	13%	29%	V2#3	Find minimal surface area of cylindrical can
42%	33%	8%	V2#5	Estimate integral given for value at five points
33%	21%	8%	V2#4	Sketch curve satisfying condition in sign table
29%	29%	0%	V1#5c	Decompose vector into orthogonal components
21%	21%	0%	V1#3	Sketch transformations of trigonometric function presented graphically
13%	13%	0%	V2#7	Use Newton's Method to approximate cube root
13%	8%	0%	V2#6	Sketch graph of trigonometric function presented algebraically
8%	8%	8%	V1#6	Estimate value of integral given graph of the function
8%	8%	0%	V1#5b	Find scalar component in direction of given vector
0%	0%	8%	V2#1a	Find equation for line through point parallel to given vector

The problems are primarily sorted in this table by proportion of students who received a code of A or B, indicating t at least essentially correct. For reference, the second and third columns report the proportion of students who had th correct (A, column 2) and the proportion who made good progress (C, column 3).

The problems have been divided into four groups. At least three-quarters of the students were able to find the equation of the line through a point such that it is parallel to a given vector, solve a 3x3 system given corresponding matrix and inverse, solve a 2x2 matrix equation, or label graphs as functions of its first and second derivatives. More then a half of the students could find and integrate the sum of vectors in a plane, use a linear approximation to estimate its value, or estimate derivatives from the graph of a function.

Less then half could correctly find the minimal surface area of a cylindrical can, estimate an integral given the function value at five points, sketch a curve satisfying the conditions in a sign table or decompose a vector into orthogonal components. Under a quarter could sketch transformations of trigonometric function presented graphically or algebraically. Similarly success rates were found when students were asked to use Newton's method to approximate a cube root, estimate the value of an integral given a graph of the function, or to find the scalar component in direction of given vector. Of the 24 who took version 2, no one found the equation for a line through a point parallel to a given vector.

Mathematics Backgrounds

University records provided information about the mathematics courses that had been taken by students in these classes. The following tables report up to the four most recent mathematics courses recorded on each student's transcript. These data reveal that most students had completed at least three university mathematics classes; more than half had completed four. Almost a third of the class was recorded taking the prerequisite course, Math 229,

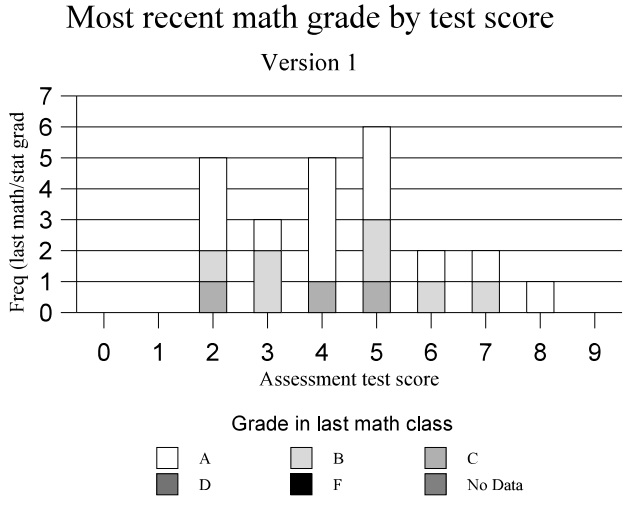
with a median grade of a B. Over three-fourths of the student have undoubtly taken the prerequisite course Math 166 with a median grade of a B.

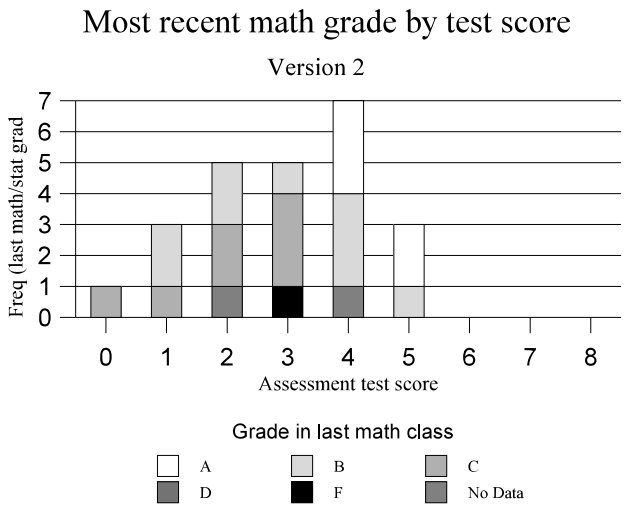
College Level Mathematics Courses (Four Most Recent Courses)

Score (max=9 V1 =8 V2)	Scoresum (max=36 V1 =32 V2)	Version	Math Courses (Course—Semester—Grade*)			
8	33	1	M229 013 A			
7	28	1	M266 013 A	M259 011 A	M229 011 B	M161 003 A
7	29	1	M266 011 B	M229 011 A	M259 003 B	M229 003 D
6	26	1	M266 011 A	M229 011 A	M259 004 A	M161 003 A
6	24	1	M265 011 B	M261 003 A	M260 001 D	M161 993 C
5	23	1	M229 011 B			
5	22	1	M266 013 A	M270 011 A	M259 011 B	M229 011 B
5	21	1	M266 011 C	M259 003 C	M229 003 C	M161 001 C
5	21	1	M266 013 A	M259 011 B	M229 011 B	M161 003 C
5	24	1	M259 011 A	M229 011 A	M161 003 A	
5	20	1	M266 001 B	M229 004 C	M259 003 B	M161 001 C
4	18	1	M266 013 A	M259 011 A	M229 011 A	M161 003 A
4	18	1	M266 013 A	M229 013 A	M259 011 A	M259 003 F
4	18	1	M266 011 A	M259 004 A	M229 004 A	M161 003 B
4	17	1	M266 013 A	M259 011 A	M229 011 A	M161 003 A
4	21	1	M266 011 C	M260 003 D	M259 003 F	M259 004 D
3	17	1	M261 004 A	M259 003 C	M229 003 A	M161 001 B
3	16	1	M261 003 B	M260 001 C	M161 993 A	M160 991 B
3	15	1	M259 013 B	M229 013 A	M166 011 B	M160 003 A
2	14	1	M266 011 A	M259 003 A	M229 003 A	
2	12	1	M266 011 A	M259 003 A	M229 011 B	M161 993 C
2	11	1	M266 013 C	M259 011 B	M229 011 B	M161 003 B
2	8	1	M160 991 B			
2	14	1	M160 991 A			
5	20	2	M266 011 B	M259 003 A	M229 033 A	M161 001 A
5	20	2	M266 011 A	M259 003 A	M229 033 A	M161 001 A
5	20	2	M261 003 A	M260 001 B	M161 993 A	M160 991 A
4	17	2	M229 004 A	M259 003 C	M161 001 C	M160 993 A
4	20	2	M261 003 B	M260 001 B	M161 993 A	M160 991 A
4	18	2	No Data Available			
4	22	2	M266 013 B	M259 011 C	M229 011 B	
4	17	2	M261 993 A	M260 991 B	M161 983 B	M160 981 A
4	17	2	M261 993 B	M260 991 B	M260 981 F	M260 973 F
4	16	2	M266 013 A	M259 011 A	M229 011 A	M161 003 A
3	15	2	M266 011 C	M259 003 B	M229 003 A	M161 993 B
3	16	2	M229 014 B			
3	15	2	M266 013 C	M259 011 B	M229 011 B	
3	14	2	M266 013 F	M266 011 F	M259 003 B	M229 003 B
3	13	2	M229 011 C			
2	11	2	M261 993 B	M260 991 C	M161 983 C	M160 981 B
2	13	2	No Data Available			
2	12	2	M229 011 B			
2	15	2	M229 011 C			
2	8	2	M229 011 C			
1	10	2	M266 013 B	M259 011 B	M229 011 B	M161 003 A
1	7	2	M266 011 C	M259 003 B	M229 003 C	M161 001 B
1	13	2	M259 011 B	M229 011 C		
0	2	2	M266 013 C	M261 003 I	M260 001 B	M161 993 C

NDSU Mathematics Courses

100-102 Pre College Algebra
103College Algebra
124Finite Math (now 104)
142Trigonometry (now 105)
146-147Business Calculus Sequence
160-161, 260Regular Calculus Sequence (now 165-166, 265, 259)
261Differential Equations (now 266)
263Vector Calculus (now in 265)
228, 229Basic Linear Algebra
329Linear Algebra





These histograms help to illustrate the connection between test score and grade in most recently completed mathematics or statistics course. As one might expect, students with higher grades (lighter shades in divided frequency bars) in most recent course generally scored somewhat higher on this assessment test.

Reactions
Near the end of the semester, the students were given the opportunity to comment about the assessment. Nine questions about the examination were asked. We also asked the instructor five questions about the test results. Their responses are summarized below.

Students: Students in the class were surveyed about the assessment test at the end of the semester. About two-thirds felt that the problems on the test did not match the quantitative skills needed for the course. Most of the students found that when they went through the corrected examination they were able to understand how to solve the problems. Almost a third of the class spent time reviewing the mathematics after taking the test (they were not informed about the exam beforehand).

Instructor: Professor Jazar felt the test results was a bit below his expectation, but overall saw a good result. After seeing the results he realized he could skip some math topics and accelerate the class so that they could cover more important topics. He agrees that the skills tested were essential for survival in this course.

Department: No reactions have been received yet from the mathematics department.

Percentages refer to the proportions of the 24 students who took the test.

1. We know for a function $y = f(x)$ that $f(3) = 5$ and $f'(3) = 2$. Use linear approximation to estimate the value of $f(3.1)$.

Used a valid method: 8%
Obtained correct value (5.2): 54%

Degree of Success: A 58% B 0% C 0% D 13% E 29%

2. Solve this system of equations:

$$\begin{cases} x + 2y + 3z = -1 \\ x + y + 2z = 3 \\ y + 2z = 4 \end{cases}$$

You may find it useful to know that

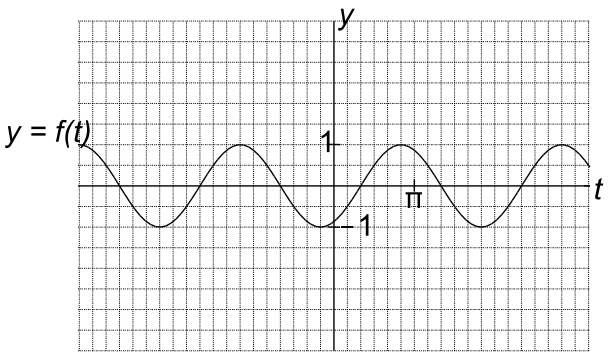
$$\begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

which would make it easy to use a matrix method to solve the system of equations.

Wrote an appropriate matrix expression or equation which would give the solution: 54%
Correctly multiplied vector by matrix to obtain the solution vector: 29%
Used a non-matrix method, such as elimination and back substitution: 29%
Used row reduction method: 54%

Degree of Success: A 88% B 0% C 0% D 13% E 0%

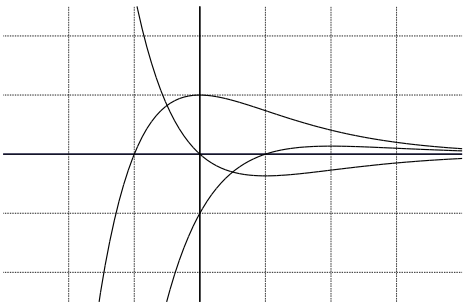
3. The graph shows a trigonometric function $y = f(t)$. On the same grid, sketch the graph of $y = 2 \cdot f\left(t - \frac{\pi}{3}\right) + 1$.



Showed (or stated) correct amplitude (2): 21%
Showed (or stated) correct period (2 pi--unchanged): 17%
Showed (or stated) correct phase shift (pi/3): 21%
Showed (or stated) correct vertical translation (1): 17%

Degree of Success: A 21% B 0% C 0% D 67% E 13%

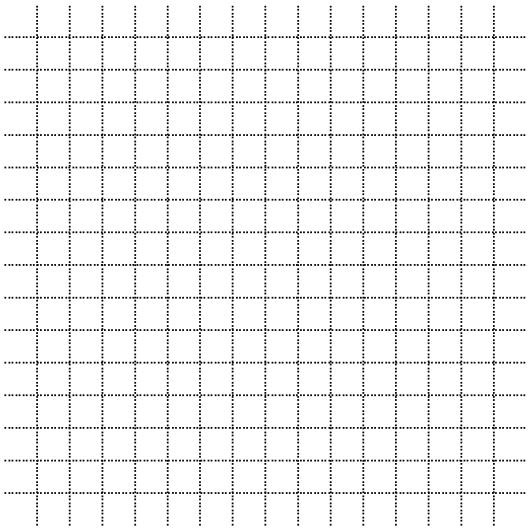
4. This figure gives the graphs of a function, f, and its first and second derivatives, f′ and f″. Label each curve as the function or its first or second derivative. Explain your answers.



Correctly labeled the three curves: 75%
Correctly related f and its first derivative (e.g. at horizontal tangents of f): 29%
Correctly related the first and second derivative: 33%
Correctly related f and its second derivative: 33%

Degree of Success: A 33% B 42% C 0% D 25% E 0%

5. (a)Find the sum of the vectors $\vec{A} = -4\vec{i} + 3\vec{j}$ and $\vec{B} = 2\vec{i} + \vec{j}$. Sketch the vector that is their sum on the grid.



Correctly found sum: 67%
Correctly sketched vector (must indicate reference coordinate system in some way): 46%

Degree of Success: A 42% B 25% C 0% D 4% E 29%

(b) Find the scalar component of $\vec{A}$ in the direction of $\vec{B}$.

Set up dot product correctly (numerator of provided formula): 4%
Set up correct expression to compute the magnitude of $\vec{\mathbf{B}}$: 4%
Found correct value for the scalar component: 8%

Degree of Success: A 8% B 0% C 0% D 50% E 42%

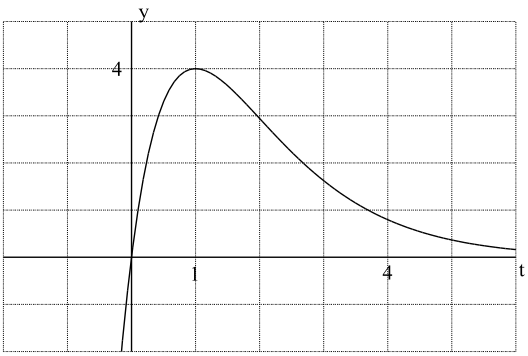
(c) Express $\vec{A} = -4\vec{i} + 3\vec{j}$ as the sum of two vectors, one parallel and the other perpendicular to the vector $\vec{B} = 2\vec{i} + \vec{j}$.

Founds direction of $\vec{\mathbf{B}}$ (unit vector in the direction of $\vec{\mathbf{B}}$): 4%
Multiplied by scalar component from (b) to get desired vector parallel to $\vec{\mathbf{B}}$: 13%
Subtracted the vector parallel to $\vec{\mathbf{B}}$ from $\vec{\mathbf{A}}$ to get the perpendicular vector: 8%
Correctly expressed $\vec{\mathbf{A}}$ as a sum of two vectors
(even if they are not orthogonal and neither is parallel to $\vec{\mathbf{B}}$): 29%

Degree of Success: A 29% B 0% C 0% D 42% E 29%

6. Estimate the value of this integral as accurately as possible from the graph of $y = f(t)$:

$$\int_0^4 f(t)\,dt$$



Used grid to estimate area:	46%
Used rectangles to estimate value:	8%
Used trapezoid rule to estimate value:	4%
Used Simpson's rule to estimate value:	0%
Method used not apparent:	100%
Gave reasonable estimate:	58%

Degree of Success: A 8% B 0% C 8% D 58% E 25%

7. Solve the matrix equation $\begin{bmatrix} -2 & 3 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 \\ -4 \end{bmatrix}$.

Attempted to use inverse matrix:	0%
Attempted row reduction:	42%
Wrote as a system of equations and attempted substitution or elimination:	25%
Correctly used appropriate method but made a computational error:	8%

Degree of Success: A 75% B 0% C 0% D 21% E 4%

Percentages refer to the proportions of the 24 students who took the test.

1.(a)Find equations for the line containing the point (−1,2,1) parallel to the vector $2\vec{i} - \vec{j} + 3\vec{k}$.

Gave parametric equations (even if incorrect): 8%
Used direction numbers from given vector (or scalar multiples of them):0%
Used given point correctly: 8%

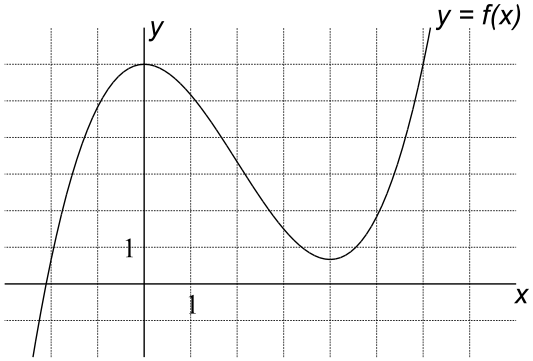
Degree of Success: A 0% B 0% C 8% D 50% E 42%

(b)Find the magnitude (or length) of $2\vec{i} - \vec{j} + 3\vec{k}$.

Set up appropriate expression for the magnitude*: 88%*

Degree of Success: A 92% B 0% C 0% D 0% E 8%

2. Here is the graph of a function $y = f(x)$. Use the graph to answer these questions:



(a) Estimate $f'(4)$.

Sketched tangent line at each point (not necessary): 21%
Commented on local minimum and/or horizontal tangent at x = 4: 38%
Stated m = 0: 54%

(b) Estimate $f'(2)$.

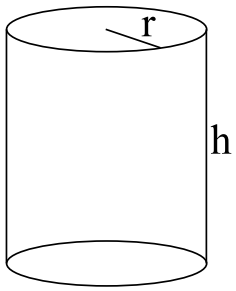
Observed that gradient at x = 2 is negative: 71%
Estimated at x=2, -3 < m < -1: 33%

(c) On which interval(s), if any, does it appear that $f'(x) < 0$?

Gave correct estimate for interval: 67%

Degree of Success: A 21% B 38% C 21% D 17% E 4%

3. A cylindrical aluminum can (see figure) is to contain 500 ml. (cubic centimeters) of apple juice. These formulas may be useful as you answer the following questions:



Volume of cylinder = $\pi r^2 h$
Area of a rectangle = length x width
Area of circle = πr^2
Circumference of circle = $2\pi r$

(a) Express h, the height of the can, as a function of r, the radius of the circular top of the can.

Correctly expressed h as a function of r: 79%

(b) Express A, the surface area of the can including the side and the circular top and bottom, as a function of r, the radius of the top.

Exhibited rule for area of side of can: 75%
Exhibited rule for area of top and bottom of can: 79%
Derived function for surface area in terms of the radius: 8%

(c) Find the radius that gives the smallest possible surface area for this can.

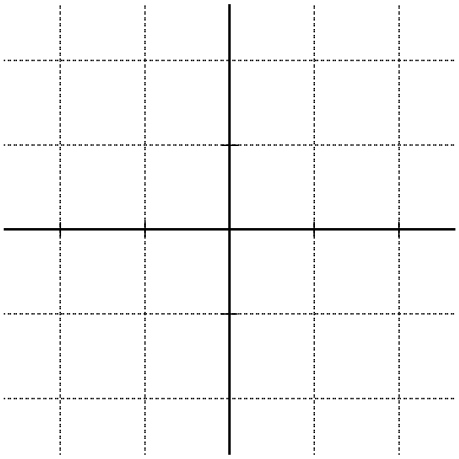
Found derivative of function from question 3b: 33%
Attempted to find the zero of the derivative: 21%
Found the zero (any exact or approximate form of the answer is alright): 4%
Confirmed this is a local minimum, e.g. used second derivative
(any valid method is coded yes): 0%

Degree of Success: A 13% B 33% C 29% D 21% E 4%

4. Here is a sign chart for a function, $y = f(x)$, and its first and second derivatives, f' and f''.

	x < -1	x = -1	-1 < x < 1	x = 1	x > 1
f	+	undefined	+	0	-
f'	+	undefined	-	0	-
f''	+	undefined	+	0	-

(a) Sketch the graph for a function satisfying the conditions in this table.



Appropriate graph for x < -1: 46%
Appropriate graph for -1 < x < 1: 25%
Appropriate graph for 1 < x: 42%

(b) For which values of *x* is the function *decreasing*?

Gave correct interval where f is decreasing: 63%

Degree of Success: A 21% B 13% C 8% D 42% E 17%

5. This table gives the values of a function $y = f(x)$ at five points. Assume that the function is continuous over the interval [0,4] and use the table to approximate

$$\int_0^4 f(x)\,dx$$

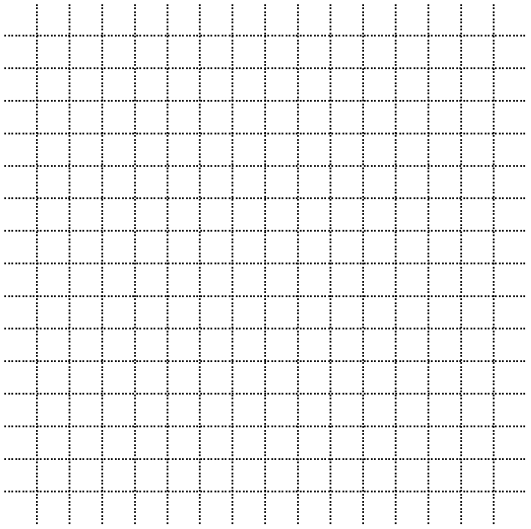
SHOW YOUR WORK.

x	0	1	2	3	4
f(x)	2	3	2	2	6

Sketched graph (not necessary): 63%
Used rectangles to estimate value: 17%
Used trapezoid rule to estimate value: 4%
Used Simpson's rule to estimate value: 17%
Method used not apparent (or if other, describe): 67%
Gave reasonable estimate: 13%

Degree of Success: A 33% B 8% C 8% D 17% E 33%

6. Sketch a graph of the function $y = A\cos(\theta - B\pi) + C$. Assume that $A, B, C > 0$.



Sketched graph has shape of cosine curve: 42%
Correct amplitude is indicated (A): 13%
Correct phase shift is indicated (B pi): 21%
Correct vertical translation is indicated (C): 8%
Graph had correct period (2 pi): 8%

Degree of Success: A 8% B 4% C 0% D 63% E 25%

7. Find an approximate value for $\sqrt[3]{8.1}$ using Newton's method. Recall that Newton's method uses the recursive or iterative formula $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$. Use three iterations of Newton's method with an initial guess of $x_0 = 2$.

Wrote correct function: f(x)=x^3-8.2: 13%
Found f' correctly: 13%
Wrote correct expression for x_n: 13%
Found correct value for x_1: 13%
Found correct value for x_3: 13%

Degree of Success: A 13% B 0% C 0% D 17% E 71%