Assessment in Mathematics Courses, Fall 1994-95

441: Introduction to Modern Algebra

541: Modern Algebra

521: Advanced Calculus

Three upper division mathematics courses participated in an assessment during the Fall 1994 semester. The purpose was to see whether the methodology used for assessing quantitative skills of emerging juniors could provide useful information about the department's undergraduate majors.

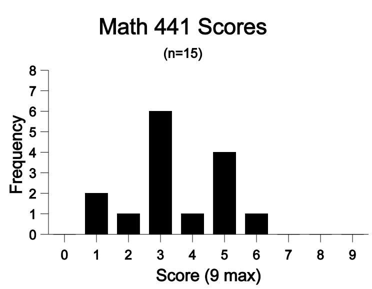
Sixteen students took a nine-item free-response test in Math 441 (Professor Benkart) Thursday, September 8, 1994. Thirty-seven students took an eight-item free-response test in Math 541 (Professors Ram and Solomon) Friday, September 9, 1994. The 441 and 541 tests had six common problems. Seventeen students submitted solutions for a seven-item free-response test in Math 521 (Professor Shea) within a week (thirty-eight students were given copies of the test). The test was handed out Thursday, September 15, 1994. Students in the class were asked to complete the test at home in an hour without reference to outside materials.

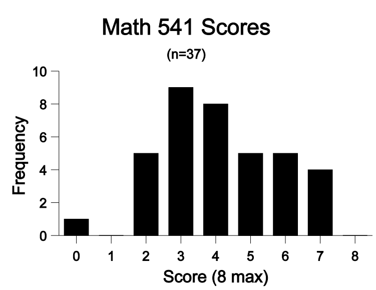
The tests were designed to see the extent to which students had basic mathematical skills required for success in the courses. Graduate students in the Department of Mathematics graded the papers, recording information about steps students had taken when solving the problems. The graders also coded the degree of success achieved on each problem using the following rubric:

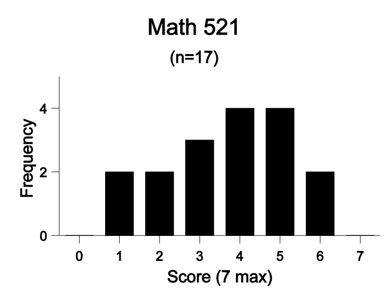
- A Completely correct
- B Essentially correct—student shows full understanding of solution and only makes a minor mistake (e.g., wrong sign when calculating a derivative or arithmetic error)
- C Flawed response, but quite close to a correct solution (appears they could do this type of problem with a little review or help)
- D Took some appropriate action, but far short of a solution
- E Blank, or nothing relevant to the problem

Corrected papers, along with suggested solutions to the problems, were returned to students within a week. Summaries of the graders' coding are included on the attached copies of questions used on the tests.

A test score was computed by awarding one point for each A or B code and zero points for each C, D, or E code. That score reflects the number of problems that a student had essentially or completely correct. The distributions of test scores are shown below.







Problems are ranked according the degree of success students achieved on each problem in the following table. Results for duplicate problems on the 441 and 541 tests were kept together and highlighted (they were sorted using the 541 success rates).

Degree of Success on Test Problems

%AB	%A	%C	Course	No	Problem description
94.1	82.4	5.9	521	2	Sketch a possible graph for a function using data about the function and derivatives in a s
86.5	83.8	10.8	541	2	"Write down all the different sequences of the digits 1,2,3,4 using each digit once and onl
78.4	64.9	13.5	541	7	List integers from -50 to 50 divisible by seven and five; and integers divisible by seven or
68.8	62.5	18.8	441	7	List integers from -50 to 50 divisible by seven and five; and integers divisible by seven or
64.7	52.9	23.5	521	1	Identify the graphs of a function and its first two derivatives and support the choices
62.5	56.3	6.3	441	8	Show that two functions given by a table of values do not commute under composition
59.5	54.1	16.2	541	3	Show that an equation with rational polynomials is an identity for real numbers other than
56.3	43.8	0	441	9	Show that an equation with rational polynomials is an identity for real numbers other than
54.1	37.8	10.8	541	1	"If A and B are sets, describe how you could prove that A≠B."
43.8	12.5	6.3	441	3	"If A and B are sets, describe how you could prove that A≠B."
47.1	47.1	35.3	521	5	Say whether two planar regions, each described algebraically, satisfy a statement
47.1	35.3	35.3	521	4	Say whether three statements about functions are true and give reasons for their answers
43.2	32.4	27.0	541	6	Write the product of complex numbers (rectangular form) in polar form
31.3	12.5	43.8	441	6	Write the product of complex numbers (rectangular form) in polar form
41.2	41.2	29.4	521	3	Compare the growth of an integral with the growth of x as x increases to infinity
41.2	35.3	0	521	7	Express a limit of a sum as a definite integral and use the integral to evaluate another limi
40.5	21.6	29.7	541	8	Explain which of four cards must be turned over to test the truth of a statement
37.5	12.5	0	441	2	Give examples of non zero matrices whose product is zero; explain meaning of unique in
35.3	29.4	35.3	521	6	Say for which real numbers exp(-x) > 1-x and support the answer
24.3	24.3	8.1	541	4	Explain why it is not true for real 2x2 matrices that A^2=A implies A=I or A=0.
12.5	12.5	12.5	441	4	Explain why it is not true for real 2x2 matrices that A^2=A implies A=I or A=0.
24.3	18.9	32.4	541	5	Explain how to find an integer b so that a+b=0 for given integer a and altered addition
0	0	31.3	441	5	Explain how to find an integer b so that a+b=0 for given integer a and altered addition
12.5	12.5	6.3	441	1	Give examples of two functions; one not 1-1, the other not onto, and defend the answers

The problems are primarily sorted in this table by proportion of students who received a code of A or B, indicating t at least essentially correct. For reference, the second and third columns report the proportion of students who had th completely correct (A, column 2) and the proportion who made good progress (C, column 3).

The problems fall naturally into four groups: High success (70% or more of the students gave correct answers), moderate success (47-65% were successful), low success (35-43% could handle these problems), and very low success (less than 25% of the students were successful).

Mathematics Backgrounds

University records provided information about the mathematics courses taken by students enrolled in these classes. That information provided the data reported in the following tables. One student who took the test did not report an ID and could not be matched to university records. The test score, computed by awarding one point for each A or B code and zero points for each C, D, or E code, is given in the first column (The first column in the table for

Math 541 indicates the section because one was an honors course). The second column, scoresum, is the sum of each students' degree of success codes (A=4, B=3, C=2, D=1, E=0). Each row corresponds to one student. A summary of the data from the tables is given below.

Math 441 students (13 had at least one graded math course). Median and modal grade in last two math courses was B. Most common courses among last two: 461 (9), 340 (5), 431 and 223 (3 each). Nine of the students had completed a math course during the preceding Spring semester. Considering the highest grade each student received in the last two courses, four had an A, three each had an AB or B, two had a BC and one had a C. This group appeared to have weaker mathematics backgrounds than students in either of the other two courses.

Math 541 students (33 had at least one graded math course). Median grade in last two math courses was AB; modal grade was A followed closely by B. Most common courses among last two were 340 (15), 319 and 521 (8 each), 223 (6), 431 (4), 371 and 522 (3). Twenty-three of the students had completed a math course during the preceding Spring or Summer semester. Considering the highest grade each student received in the last two courses, 16 had an A, 7 had an AB, and the remaining 10 had a B. These students had strong mathematics backgrounds according to university records.

Math 521 students (13 of the 14 who turned in a test paper and appeared in the ADP records had at least one graded math course). Median grade in the last course was A, median grade in the prior course was AB; modal grade for each was A and nine of the 13 had received at least one A in the past two courses. The most common courses among the last two were 319 and 541 (4 each), 340 and 567 (3). These students had strong mathematics backgrounds. It also appears that students with stronger backgrounds were more likely to hand in the test paper.

Two Most Recent Math Courses

Students in Math 441

Score (max=9)	Scoresum (max=36)	Classification	Course-Grade*	Semester	Course-Grade**	Semester
0**	2	BS	461 BC	S93	475 D	S93
1	8	SED	223 B	S94	319 F	SS94
1	14	SED	461 C	S93	475 BC	S94
2	13	SED	340 B	F93	461 AB	S94
3	14	E	none			
3	15	BS	431 D	F93	461 C	S94
3	17	EED	223 AB	S92	340 BC	F92
3	17	SED	321 C	S94	461 AB	S94
3	18	SED	340 A	S93	461 B	F93
3	21	SED	371 B	S94	461 B	S94
4	20	BA	473 B	F93	461 A	S94
5	22	PRS	340 C	SS93	431 A	S94
5	23	UNRS	521 B	SS93	541 B	SS93
5	24	BS	223 T*		340 T*	
6	27	EDCS	461 A	S94	431 C	SS94

*T indicates transfer credit for the course
**Student left room due to bee sting after 5 minutes

Two Most Recently Passed Math Courses
(Math 541)

Section	Score (max=8)	Scoresum (max=32)	Class	Yr	Math Courses (Course—Semester—Grade)					
	0	6	BS	4	319	S94	B	340	S94	A
H	2	15	BS	3	319	S94	B	371	S94	B
	2	15	BS	4	223	F93	B	340	S94	C
H	2	16	UNRS	9	none					
	2	16	SED	4	461	S93	AB	473	F93	AB
H	3	16	BS	3	223	F93	HA**	340	S94	A
H	3	16	ME	3	223	F93	A	340	SS94	BC
H	3	18	BA	3	223	F93	HAB	340	S94	HB
H	3	18	BS	3	223	F93	HBC	340	S94	AB
	3	18	BS	4	431	S91	BC	340	F92	B
H	3	18	IE	4	235	F92	A	321	SS93	AB
	3	18	UNRS	9	521	SS93	B	541	SS93	B
	3	19	BS	4	340	F91	C	319	F92	B
	3	20	SED	4	319	S89	B	321	SS91	AB
H	4	18	PRS	4	none					
	4	19	ECE	4	340	S92	A	431	S94	A
H	4	20	SED	4	521	F92	HAB	475	S93	A
	4	20	ECE	2	319	F93	B	340	S94	B
H	4	21	BS	4	340	F92	HA	521	F93	HB
	4	21	BA	4	441	S94	AB	525	S94	AB
H	4	22	ECE	4	521	S93	A	319	S94	A
H	5	21	BS	3	431	SS94	B	319		T*
H	5	22	BS	3	340	F93	B	521	S94	C
	5	22	BA	4	461	S94	B	522	S94	D
	5	26	LS	5	521	F92	HA	623	S93	HA
	5	26	BS	4	475	F93	AB	525	S94	B
	6	24	BS	3	223	F93	BC	340	S94	B
H	6	25	BS	3	371	S94	A	522	S94	HA
H	6	25	BS	4	541	S93	HC	490	F93	HA
	6	25	ED	5	521	F93	A	522	S94	A
H	6	26	BS	4	340	F93	A	521	S94	A
	7	26	BA	4	340	F93	AB	371	S94	AB
	7	29	BS	4	321	F93	A	322	S94	BC
H	7	29	BS	3	491	F93	A	567	S94	A
	7	30	BS	4	431	S94	A	319	SS94	A

*T indicates transfer credit for the course
**H in section or with grade indicates Honors section

Two Most Recently Passed Mathematics Courses
(Math 521)

Score (max=7)	Scoresum (max=28)	Class	Yr	Math—semester—grade*	
		E	5		
		E	5		
		LS	5		
		UNRS	9		
		ECE	3	235 T*	320 T
		BS	4	431 S92 BC	309 F92 C
		BS	3	309 F93 A	319 S94 A
		BS	4	632 S94 B	319 S94 C
		EM	4	340 F92 BC	321 F93 B
		BS	4	319 S93 AB	321 F93 BC
		BA	4	340 F93 BC	371 S94 B
		BS	4	525 F93 A	431 S94 A
		ECE	4	571 S93 AB	431 F93 AB
		BS	4	431 S94 C	441 S94 BC
		BS	4	321 F93 A	541 S94 B
		SED	4	473 F93 A	541 S94 B
		BS	4	441 F93 B	541 S94 BC
		BS	4	541 S93 C	542 F93 D
		BS	2	340 F93 C	551 S94 F
		BS	4	541 F93 D	567 S94 BC
		BS	4	541 S94 AB	571 S94 A
2	10	BS	4	319 S94 A	340 S94 A
2	15	BS	4	541 F93 B	567 S94 A
3	17	BS	4	303 F93 B	443 S94 B
3	19	BS	4	441 F93 B	541 S94 HA
3	20	BS	2	223 F93 A	340 S94 A
4	17	SED	4	461 F93 AB	441 S94 A
4	19	BS	4	541 F93 C	571 S94 AB
4	20	LS	5	551 F89 C	319 F90 C
4	21	BS	4	541 F93 HA	567 S94 A
5	22	BS	4	542 S94 B	551 S94 B
5	23	E	5		
5	24	BS	4	319 F93 HA	322 S94 A
6	23	E	5	319 S89 A	340 SS89 A
6	23	BS	4	371 S94 AB	567 S94 A

*T indicates transfer credit for course

Summary Remarks on Test Outcomes

An examination of assessment test scores and final grades in the two algebra courses (441, 541) revealed a significant positive correlation between the two variables: In each case the assessment test scores could be used to account for about a third of the variability in course grades, suggesting that the tests were reasonable indicators of mathematical capabilities necessary for success in the courses. There was a weaker positive correlation between grade in previous mathematics course and assessment test score: the weakness partly reflected the low variability of grades in the prior courses since most students had high grades (see summaries above). Naturally, many other factors that have not been considered here could influence test scores or course grades (e.g., the course, motivation and effort), so one would not expect high correlations between a few variables.

Students in the algebra classes were surveyed about the assessment test at the end of the semester. Their responses showed that most did not believe that the test helped them during the term, although more did believe (than did not) that the tests reflected skills needed for success in the course. Most felt that they understood the test solutions and believed that a student who did well in the course would also have done well on the assessment test. Few students did any review before the test, while more than a third reported studying after receiving their corrected test paper and solutions.

This was our first experience testing mathematics skills of higher level mathematics students (we did assess in Math 319 previously). Some problems were hard, but one would expect higher success rates overall because of a tighter focus on mathematics and more homogeneous backgrounds of students. In the assessment project, we rarely see more than 80% of a class correctly answers any but the most routine problems. Success rates below 50% suggest that many in the class have trouble and will require some action (e.g., review) to use the material in the class. Three of the 24 problems had high success rates of 78% or higher; 15 had success rates under 50%. To the extent that the tests contained problems that students should handle on entry to the courses, the results seem to show weaknesses in student preparation not revealed by their grades in prior mathematics courses.

A concern about this assessment procedure is whether students really “put out” for the test, given that it did not count. Based on observations of the proctor and graders, students seemed to take the tests seriously. There was nothing to suggest that students did not try to answer problems correctly to the best of their ability.

Few test items dealt with specific content or procedures from prior courses. Many problems asked students to explain or justify a response (441#1-5, 8, 9; 541#1, 3-5, 8; 521#1, 3, 4, 6). Other questions asked students to show an understanding of mathematical language and logic (e.g., 441#1, 7; 541#7, 8; 521#5). Many students had difficulty with such questions. Where might such skills be developed in lower division mathematics courses? That is, how do introductory mathematics courses prepare students to supply explanations at the start of a 500 level course?

Success rates for 541 were consistently higher, typically by a small amount (about 10 percentage points). As noted above, prior course work and classification of students show that the courses draw from different populations. A noticeable difference was that most 441 students were education majors, while few 541 students were. Seeing the lack of obvious differences in test scores and math backgrounds between students in the honors and regular sections of 541 was also interesting.

As we were finalizing the tests, several faculty members observed that the tests differed from their original intentions. Compared to our experiences in many other departments, it seemed especially difficult to pin down the quantitative capabilities that should be tested in these upper-division mathematics courses. Now that we have some results, deciding how well the tests covered important capabilities students need for success in later mathematics courses is important.

When doing exercises of this sort in other departments, we have often encountered situations where faculty have second thoughts about the relevance of original test questions. When the process has worked well, discussion of these thoughts has lead to better formulations of faculty desires and expectations. In such cases, departments have found the whole process helpful.

An advantage of this assessment process is that it encourages faculty reflection on specific goals of their instruction. Designing these tests for mathematics courses was perhaps more challenging than in other “client disciplines” because the prerequisite skills deal more with ways of thinking, perspectives, or mathematical maturity than specific content. Still, trying to come up with specific problems that fit a given course always keeps the discussion focused. Thinking about what the results mean also promotes reflection on goals, even if one decides that the questions were flawed or inappropriate. Student outcomes on specific problems are just one aspect of this form of assessment; perhaps they are not even the most important result.

Test Problems with Success Rates
Percentages reflect the proportion of the students in each course who took the test.

441#1. (a) Give an example of a function from the reals to reals that is not 1-1. Demonstrate that the function you give is not 1-1.

Correctly gave a non 1-1 function: 44%
Showed the function was not 1-1: 44%

(b) Give an example of a function from the reals to reals that is not onto. Demonstrate that the function you give is not onto.

Correctly gave a non onto function: 19%
Showed the function was not onto: 13%

Degree of Success:	A	13%	B	0%	C	6%	D	63%	E	19%

441#2. (a) If possible, give an example of two non zero 2×2 real matrices whose product is the zero matrix. If this is not possible, give a reason why it cannot be done.

Gave a correct pair of matrices: 56%
Said it was not possible: 25%

(b) What does it mean to say that the inverse of a real 2×2 matrix is unique?

Gave a valid explanation: 50%

Degree of Success:	A	13%	B	25%	C	0%	D	56%	E	6%

441#3. If A and B are sets, describe how you could prove that A ≠ B.

Degree of Success:	A	13%	B	31%	C	6%	D	19%	E	31%

541#1.

Degree of Success:	A	38%	B	16%	C	11%	D	24%	E	11%

441#4. If one considers the set of real numbers $\mathbb{R}$, then $a \in \mathbb{R}$ **and** $a^2 = a$ forces a to be 0 or 1. If $A \in \mathbb{R}^{2\times 2}$ (the set of 2×2 matrices with real entries) and $A^2 = A$, is it true that $A = 0$ **or** $A = I$? Why or why not?

Stated that this is not true: 38%
Gave a valid counter example: 13%
Gave a valid explanation: 0%

Degree of Success: A 13% B 0% C 13% D 44% E 31%

541#4. Stated that this is not true: 32%
Gave a valid counter example: 24%
Gave a valid explanation: 0%

Degree of Success: A 24% B 0% C % D 57% E 11%

441#5. Suppose that we make a rule that we will no longer carry digits when we add so that, for example:

$$\begin{array}{r} 9 \\ +6 \\ \hline 5 \end{array} \qquad \begin{array}{r} 175 \\ +986 \\ \hline 51 \end{array}$$

Suppose you are given a positive integer a. Describe how to find another positive integer b such that a + b = 0.

Demonstrated understanding of the problem: 19%
Described a valid strategy for finding b: 0%
Described a flawed strategy that worked sometimes: 25%

Degree of Success: A 0% B 0% C 31% D 31% E 38%

541#5.
Demonstrated understanding of the problem: 57%
Described a valid strategy to find b: 24%
Gave a flawed strategy that worked for some cases: 35%

Degree of Success: A 19% B 5% C 32% D 30% E 14%

441#6. Simplify $(\sqrt{3}+i)\cdot(2+2\sqrt{3}i)$. Write the product in the polar form $re^{i\square}$.

Multiplied correctly to obtain 8i: 81%
Correct value for r in polar form: 75%
Correct value for theta in polar form: 31%

Degree of Success: A 13% B 19% C 44% D 19% E 6%

541#6.
Multiplied correctly to obtain 8i: 76%
Found correct value for r in polar form: 65%
Found correct value for theta in polar form: 54%

Degree of Success: A 32% B 11% C 27% D 27% E 3%

441#7. (a) List all of the integers between -50 and 50 that are divisible by either seven or five.

Correctly gave all multiples of 5: 88%
Correctly gave all multiples of 7: 88%
Only left out 0: 19%

(b) List all of the integers between -50 and 50 that are divisible by both seven and five.

Correctly gave all multiples of 35: 88%
Only left out 0: 13%

Degree of Success: A 63% B 6% C 19% D 0% E 13%

541#7. (a)
Correctly gave all multiples of 5: 92%
Correctly gave all multiples of 7: 92%
Only left out zero: 8%
(b)
Correctly gave all multiples of 35: 95%
Only left out zero: 11%

Degree of Success: A 65% B 14% C 14% D 8% E 0%

441#8. This table defines two functions, f and g, that map the set {a, b, c} to itself. Is it true that $f \circ g = g \circ f$? ($\circ$ denotes composition of functions) Support your answer.

x	f(x)	g(x)
a	a	b
b	b	b
c	b	a

Correctly said the functions do not commute: 81%
Gave a valid reason for their answer: 63%

Degree of Success: A 56% B 6% C 6% D 13% E 19%

441#9. Is the following equation satisfied by every real number x other than 3 and −2?

$$\frac{x^3 + 3x^2 - 4x - 12}{x^2 - x - 6} = x + 4 + \frac{6}{x - 3}$$

You must support your answer.

Correctly stated that the equation does hold: 56%
Gave a valid justification for their answer: 56%
Correctly factored left side of equation: 44%
Correctly showed that reduced left was equivalent to right: 44%

Degree of Success: A 44% B 13% C 0% D 31% E 13%

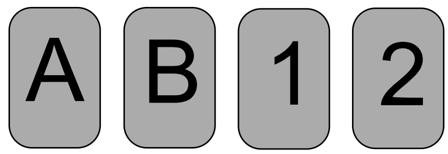
541#3.
Correctly stated that the equation does hold: 84%
Gave a valid justification: 60%
Correctly factored left side: 46%

Degree of Success: A 54% B 5% C 16% D 19% E 5%

541#2. Write down all the different sequences of the digits 1, 2, 3, 4, using each digit once and only once.

Degree of Success: A 84% B 3% C 11% D 0% E 3%

541#8. Four cards have a digit printed on one side and a letter printed on the other side. Suppose the cards are lying on the table as shown:

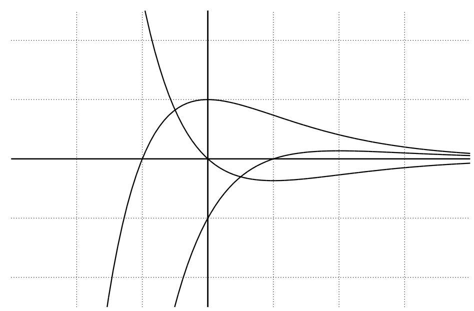


Exactly which card or cards must be turned over to test the truth of the statement: "Cards with vowels printed on one side have an even number printed on the other side." Explain why your choice is correct.

Correctly said A should be turned: 97% Gave valid reason: 89%
Correctly said B need not be turned: 95% Gave valid reason: 46%
Correctly said 1 should be turned: 60% Gave valid reason: 57%
Correctly said 2 need not be turned: 70% Gave valid reason: 30%

Degree of Success: A 22% B 19% C 30% D 30% E 0%

521#1. This figure gives the graphs of a function, f, and its first and second derivatives, f' and f''. Label each curve as the function or its first or second derivative. Explain your answers.



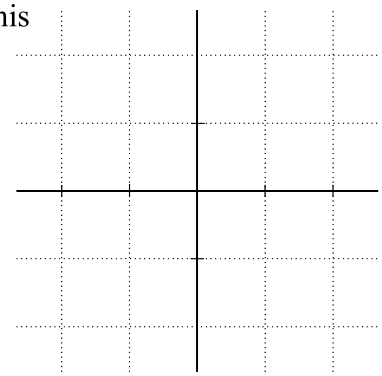
Correctly labeled the three curves: 71%
Correctly related f and f': 71%
Correctly related f' and f'': 41%
Correctly related f and f'': 47%

Degree of Success: A 53% B 12% C 24% D 6% E 6%

521#2. Here is a sign chart for a function, $y = f(x)$, and its first and second derivatives, f' and f''.

	$x < -1$	$x = -1$	$-1 < x < 1$	$x = 1$	$x > 1$
f	+	undefined	+	0	–
f'	+	undefined	–	0	–
f''	+	undefined	+	0	–

(a) Sketch a possible graph for a function that satisfies the conditions in this table.



Appropriate graph for $x < -1$: 100%
Appropriate graph for $-1 < x < 1$ 100%
Appropriate graph for $1 < x$: 94%

(b) For which values of x is the function decreasing?

Gave correct interval: 88%

Degree of Success: A 82% B 12% C 6% D 0% E 0%

521#3. As $x \to \infty$, does $\int_1^x \ln(t)\,dt$ grow faster than x, slower than x, or at essentially the same rate as x? Why?

Correctly stated that the integral grows faster than x: 53%
Gave a valid reason to support correct answer: 47%

Degree of Success: A 41% B 0% C 29% D 18% E 12%

521#4. True or false? Why?

(a) For all $x \geq 4$, $\sqrt{x} \geq 2\ln(x)$.

Correctly said this is false: 94% Valid support for a correct response: 71%

(b) For all $x \geq 4$, the graph of $f(x) = 2x + \sin x$ is always increasing.

Correctly said this is true: 82% Valid support for a correct response: 77%

(c) For all $x \geq 4$, the graph of $f(x) = 8x + \sin(x^2)$ is always increasing.

Correctly said this is false: 88% Valid support for a correct response 53%

Degree of Success: A 35% B 12% C 35% D 18% E 0%

521#5. Here is a statement:

> S is a set in the plane such that for any two points P_1 and P_2 in S, the line segment $\overline{P_1P_2}$ lies entirely in S.

Examples: The first set sketched satisfies the statement. The second does not.

Which of the following satisfy the statement?

(a) S is the set of points (x,y) such that $x^2 + y^2 \geq 3$.

Correctly said this set does not satisfy the statement: 100% Gave a valid reason: 71%
Showed correct region: 65%

(b) S is the set of points (x,y) such that $x \leq y \leq x^2$.

Correctly said this set does not satisfy the statement: 53% Gave a valid reason: 53%
Showed correct regions: 53%

Degree of Success:	A	47%	B	0%	C	35%	D	12%	E	6%

521#6. For what real numbers x is this inequality true? Justify your assertion.

$$e^{-x} > 1 - x$$

Correctly stated all non zero real numbers 53%
Supported their correct response: 35%

Degree of Success:	A	29%	B	6%	C	35%	D	29%	E	0%

521#7. Let $f(x)$ be a continuous function. Express

$$\lim_{n \to \infty} \frac{1}{n}\left[f(1/n) + f(2/n) + \cdots + f(n/n)\right]$$

as a definite integral. Use your answer to evaluate the limit

$$\lim_{n \to \infty} \frac{1}{n^4}\left[1^3 + 2^3 + 3^3 + \cdots + n^3\right]$$

Gave a correct definite integral:	41%
Correctly represented second limit as an integral:	41%
Correctly evaluated the second limit:	41%

Degree of Success: A 35% B 6% C 0% D 18% E 41%