Quantitative Readiness for Food Science 440 Name_____SOLUTIONS_____
(version 1, Fall 1993) SHOW ALL WORK!

References are to (C) Thomas & Finney, *Calculus and Analytic Geometry, 7th Edition* and (P) Cohen, *College Algebra, 2nd Edition*.

You may find these formulas useful on the test:

For a circle with radius r: Circumference = $2\pi r$ Area = πr^2

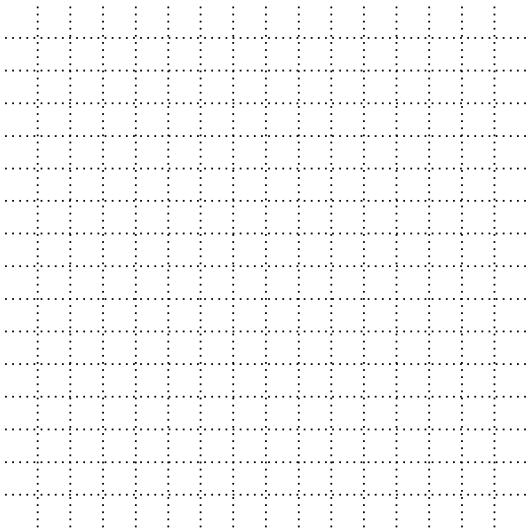
The derivative of the exponential function $y = e^u$ is given by $\frac{dy}{dx} = \frac{du}{dx} e^u$.

You may use any type of calculator on this test

1. (a) Sketch the graph of this system of equations on the provided grid:

$$\begin{cases} x - y = 1 \\ x - 3y = -15 \end{cases}$$

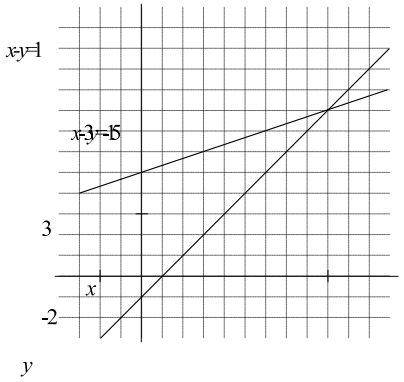
(b) Solve the system of equations given in part (a).



(a) Refer to the graph.

(b) Eliminating x gives the equation $2y = 16$, so $x = 9$ and then $y = 8$.

(P) Chapters 2, 7



2. (a) The formula $F = \frac{9}{5}C + 32$ can be used to convert temperatures from celsius to fahrenheit. Solve this formula for C in terms of F.

(b) Convert the temperature 95° F to its equivalent in degrees celsius.

(a) $C = \frac{5}{9}(F - 32)$

(b) $C = \frac{5}{9}(95 - 32) = \frac{5}{9} \cdot 63 = 35$ (P) Section 2.1

3. Given that $\hat{p}_1 = 0.50,\ \hat{p}_2 = 0.30,\ n_1 = 100,$ and $n_2 = 100$, evaluate:

$$(\hat{p}_1 - \hat{p}_2) + 1.96\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

$$(0.50 - 0.30) + 1.96\sqrt{\frac{0.50(1-0.50)}{100} + \frac{0.30(1-0.30)}{80}}$$ (P) Section 1.6

$$= 0.20 + 1.96\sqrt{0.0025 + 0.002625} \approx 0.340$$

4. The number of bacteria in a culture doubles every eight hours. The number of bacteria present, A, after t hours is given by the formula

$$A = 500 \times 2^{\frac{t}{8}}$$

(a) How many bacteria are present after 48 hours?

(b) After how many hours will the culture contain at least 4000 bacteria?

(a) $A = 500 \times 2^{\frac{48}{8}} = 32{,}000$ (b) Solve $4000 = 500 \times 2^{\frac{t}{8}}$: $\frac{4000}{500} = 2^{\frac{t}{8}}$, so

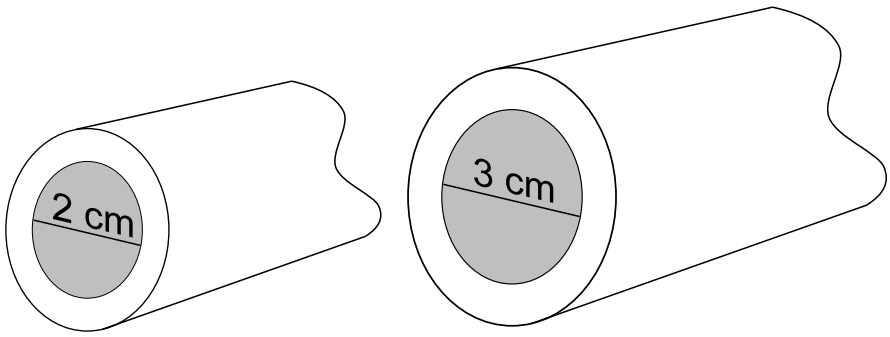
$\log_2 8 = \frac{t}{8} \Rightarrow t = 8 \times 3 = 24$ hours. (P) Chapter 6

5. Does the function $y = e^{3x}$ satisfy the differential equation $\frac{dy}{dx} = 3y$? You must support your answer.

$\frac{dy}{dx} = 3e^{3x} = 3y$ so $y = e^{3x}$ is a solution of the differential equation.

(C) Section 4.1; Sections 20.1, 20.2

6. A plumber needs to compare the capacity of two circular pipes with diameters 2 cm and 3 cm. The capacity is proportional to the circular cross sectional area of the pipe.



(a) Calculate the cross sectional area of each pipe.

(b) Express the cross sectional area of the 2 cm pipe as a percentage of the cross sectional area of the 3 cm pipe.

(a) $A = \pi r^2 = \pi, \ \frac{9}{4}\pi$

(b) The ratio is $\frac{\pi}{\frac{9}{4}\pi} = \frac{4}{9}$, so the 2 cm pipe has approximately 44% of the capacity of the 3 cm pipe.

(P) Section 3.3

7. Sound pressure (intensity) varies inversely as the square of the distance from the source. (The sound pressure is measured by a logarithmic scale with units decibels (db). A decrease of 3 db corresponds to a halving of the sound pressure.)

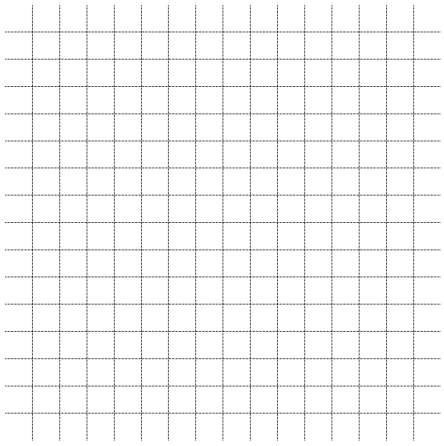
Distance (meters)	1	2	3	4	5
Sound pressure (db)	97	91		85	

The table gives the sound pressure at various distances for a source with sound intensity of 97 db at one meter. Use the table to estimate these values (use linear interpolation for parts (a) and (b)):

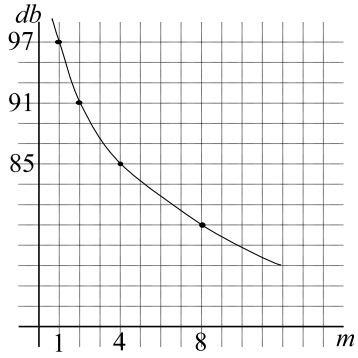
(a) What is the sound pressure at 3 meters distance from the source?

(b) At what distance would the sound pressure be 96 db?

(c) Use a sketch graph or some other suitable method to estimate the sound pressure at a distance of 8 meters.

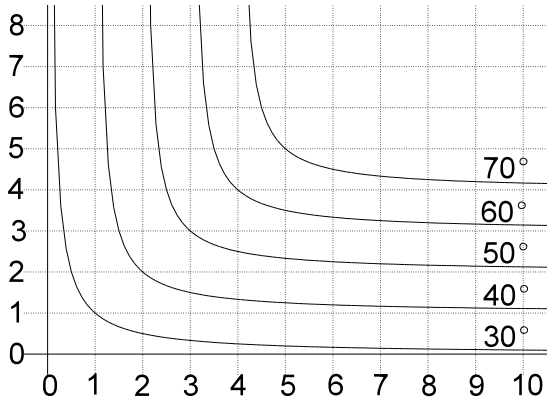


(a) Using linear interpolation, 88 db (half way between values in the table)
(b) Using linear interpolation, 1.17m (1/6th the distance from 1 to 2)
(c) It will be 1/4 the intensity of 4 meters, or 6 db less than 85db: 79 db.



(P) Section 3.3

8. This graph shows 10 degree temperature isolines over an area. For example, the temperature at the position (3,3) is 50°.



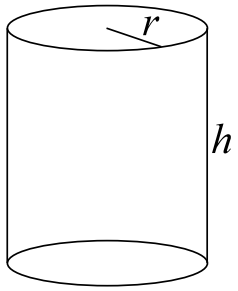
(a) What is the temperature at the position (4,4)?

(b) Estimate the temperature at the position (4,2).

(c) Estimate the temperature at the position (1.5,3).

(a) The temperature is 60°.
(b) The temperature is about 46° (or 47°, using interpolation)
(c) The temperature is about 40° (using interpolation to find the position).
Answers for (b) and (c) should be within 1 degree. (P) Sections 3.1, 4.2

9. Find the surface area of a cylindrical can, including its circular top and bottom. The can has a diameter of 10 cm and a height of 18 cm. Several useful formulas are given at the beginning of this test.



Area of the circular top (and bottom): $A_C = \pi r^2 = 5^2\pi = 25\pi$

Area of rectangular side (length is circumference of the circular top):

$$A_R = l \times h = 2\pi rh = 10\pi \times 18 = 180\pi$$

Total surface area: $2A_C + A_R = 50\pi + 180\pi = 230\pi \approx 723 \text{ cm}^2$. (P) Section 2.3