NAME________________

SHOW ALL WORK!

You may use any type of calculator on this test.

1. Given that $p_1 = 0.50$, $p_2 = 0.30$, $n_1 = 100$, and $n_2 = 80$, evaluate:

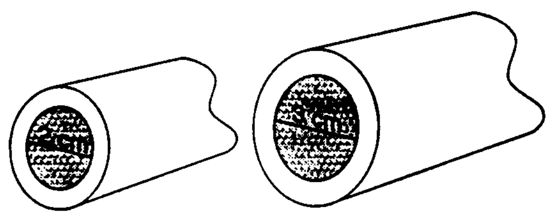
$$(p_1 - p_2) + 1.96\sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$$

$$(0.5 - 0.3) + 1.96\sqrt{\frac{(0.5)(0.5)}{100} + \frac{(0.3)(0.7)}{80}}$$

$$(0.2) + 1.96\sqrt{0.0025 + 0.002625}$$

$$0.2 + 1.96(0.005125) = \boxed{0.210045}$$

2. A plumber needs to compare the capacity of two circular pipes with diameters 2 cm and 3 cm. The capacity is proportional to the circular cross sectional area of the pipe.



(a) Calculate the cross sectional area of each pipe.

2 cm

$A = \pi r^2$

$r = \frac{1}{2} d$

$A = \pi (1\ cm)^2$

$\boxed{A = 3.14\ cm^2}$

3 cm

$A = \pi r^2$

$r = \frac{1}{2} d$

$A = \pi (1.5\ cm)^2$

$A = 3.14\ (2.25)\ cm^2$

$\boxed{= 7.065\ cm^2}$

(b) Express the cross sectional area of the 2 cm pipe as a percentage of the cross sectional area of the 3 cm pipe.

$3.14 / 7.065 = 0.4444$

The cross sectional area of the 2 cm pipe is 44.44% the area of the 3 cm pipe.

3. Does the function $y = e^{3x}$ satisfy the differential equation $\frac{dy}{dx} = 3y$? You must support your answer.

$\ln y = 3x$

Yes, the natural log of both sides must be take before differentiation takes place. this gives the equation $\ln y = 3x$ Upon differentiation you get $\frac{dy}{dx}\,{}^1\!/y = 3$, By rearrangement, you get $\frac{dy}{dx} = 3y$.

4. The number of bacteria in a culture doubles every eight hours. The number of bacteria present, A, after t hours is given by the formula

$$A = 500 \times 2^{\frac{t}{8}}$$

(a) How many bacteria are present after 48 hours?

$A = 500 \times 2^{48/8}$

$= 500 \times 2^6$

$= 500 \times 64 = \boxed{32{,}000 \text{ bacteria}}$

(b) After how many hours will the culture contain at least 4000 bacteria?

$4000 = 500 \times 2^{t/8}$

$8 = 2^{t/8}$

$\boxed{t = 24 \text{ hours}}$

$2^3 = 8$ $8 \times 3 = 24$

$8 = 2^{24/8}$

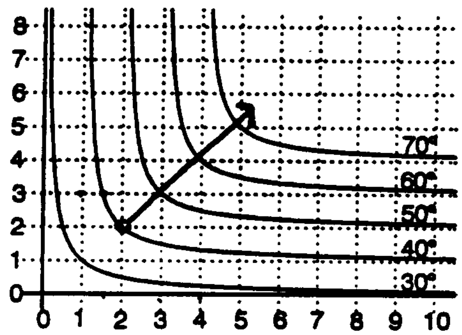
$2^3 = 8$

5. In this differential equation F and V are constants: $0 = Fx + V\frac{dx}{dt}$. Separating variables it can be written in differential form as $\frac{dx}{x} = -\frac{F}{V}dt$. Integrate each side of this form of the equation to find an explicit solution that gives x as a function of t.

$$\int 0 = F_x + V\frac{dx}{dt}$$

$$\boxed{0 = \frac{F}{2}x^2 + Vx}$$

6. This graph shows 10 degree iso temperature lines over an area. For example, the temperature at position (3,3) is 50°.



(a) What is the temperature at the position (4,4)?

60°

(b) Estimate the temperature at the position (4,2).

45°

(c) Estimate the temperature at the position (1.5,3).

40°

(d) Give the coordinates for two different positions with a temperature of 35°.

(3, 1) and (1.5, 2)

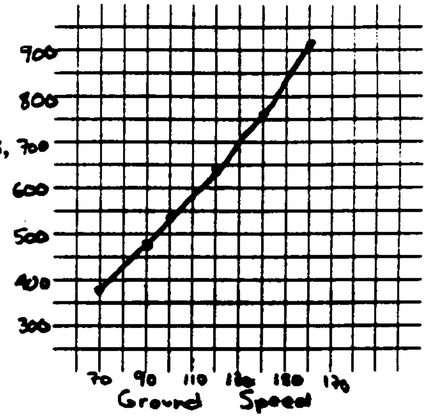
(e) Draw an arrow at (2,2) that points in the direction of the most rapid increase of temperature.

ON Graph

7. Commercial airline pilots landing at the Los Angeles International Airport approach the runway along a glide slope that makes a 3° angle with the runway. They can use this table to manually maintain the approach angle. The table relates the aircraft's rate of descent, in feet per minute, to its ground speed, measured in knots.

Ground Speed (knots)	70	90	100	120	140	160
Rate of Descent (fpm)	379	487	541	649	757	866

Use linear interpolation to estimate the following values. [You may wish to plot the values in the table to help with this problem, though it is not necessary to do so.]



(a) If an aircraft is approaching the runway at a ground speed of 110 knots, what rate of descent (in fpm) will maintain the appropriate glide slope?

$\frac{70}{379} = \frac{110}{x}$ $70x = (110)(379)$

$\boxed{x = 595 \text{ fpm}}$

(b) An aircraft is descending at a rate of 406 fpm. What ground speed should the pilot maintain to approach the runway at the correct angle?

$\frac{70}{379} = \frac{x}{406}$ $379x = (70)(406)$

$\boxed{x = 75 \text{ knots}}$

8. A cleaning solution is made up by combining one part phosphoric/glycolic acid concentrate with eight parts of water. A food technician has 10 liters of the cleaning solution already prepared and needs to increase this to 25 liters.

(a) How many liters of acid concentrate and how many liters of water should be added to the existing 10 liters of cleaning solution to obtain 25 liters at the correct dilution?

$\frac{1}{8}$

15 x 1/8 = 1.875 liters of acid

15 x 7/8 = 13.125 liters of H_2O

(b) The acid concentrate is a 10% solution (i.e., it is 90% water, 10% acid). What is the concentration of the diluted solution?

25 x 1/8 = 3.125 Liters acid

3.125 x 0.10 = 0.3125 liters acid

$\frac{0.3125}{25} = \boxed{1.25}$